

Integration by substitution

1. Evaluate:

(a) $\int e^{ax} 2^x dx$

(b) $\int (2^x + 3^x)^2 dx$

(c) $\int \frac{dx}{x \ln x \ln(\ln x)}$

(d) $\int \frac{dx}{e^x + e^{-x}}$

(e) $\int \frac{e^x dx}{ae^x + b}$

(f) $\int \frac{e^x dx}{\sqrt{e^x + 1}}$

(g) $\int \frac{e^{\sqrt{x}} - 3}{\sqrt{x}} dx$

(h) $\int \frac{\ln 2x}{x} dx$

(i) $\int e^{\sin x} \cos x dx$

(j) $\int e^x (1 - e^x)^n dx$

(k) $\int \frac{dx}{x(1 + \ln x)^2}$

(l) $\int_0^{\sqrt{\ln 2}} x^3 e^{-x^2} dx$

(m) $\int_0^{1/2} x \ln \frac{1+x}{1-x} dx$

(n) $\int \frac{2^x 3^x}{9^x - 4^x} dx$

(o) $\int \sec 3x \tan 3x dx$

(p) $\int \sec^2(3x + 7) dx$

(q) $\int \frac{\cos x}{\sin^2 x} dx$

(r) $\int \cos^2 3x \sin 2x dx$

(s) $\int \tan \frac{x}{2} dx$

(t) $\int \frac{\cos x dx}{1 + 2 \sin x}$

(u) $\int x \sin(x^2) dx$

(v) $\int \tan^2 x \sec^2 x dx$

(w) $\int (\cos x + 2 \sin x)^2 dx$

(x) $\int x \csc^2 x^2 dx$

(y) $\int (\tan x + \cot x)^2 dx$

(z) $\int \frac{\cos x dx}{\sqrt{a + b \sin x}}$

2. Evaluate:

(a) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

(b) $\int \sin x \sin 3x dx$

(c) $\int \sin 2x \cos x dx$

(d) $\int \tan \sqrt{1+x^2} \frac{xdx}{\sqrt{1+x^2}}$

(e) $\int \frac{dx}{A^2 \sin^2 x + B^2 \cos^2 x}$

(f) $\int \frac{dx}{\sin x \cos x}$

(g) $\int \frac{dx}{\sin^2 \left(x + \frac{\pi}{4} \right)}$

(h) $\int \frac{\sin x \cos x}{1 + \sin^4 x} dx$

(i) $\int \frac{\sin x + \cos x}{\sqrt[3]{\sin x - \cos x}} dx$

(j) $\int \frac{x^2 - 1}{x^4 + 1} dx$

(k) $\int \sqrt{\frac{a+x}{a-x}} dx$

(l) $\int_{-\pi/4}^{\pi/4} \tan x dx$

(m) $\int_{-1/5}^{1/5} x \sqrt{2 - 5x} dx$

3. Evaluate $\int \frac{x^3 dx}{\sqrt{x^2 - a^2}}$ by substituting $t = \sqrt{x^2 - a^2}$.

4. Evaluate:

(a) $\int \frac{dx}{(x+1)(x+2)(x+3)}$

(b) $\int \frac{dx}{x^3 + 1}$

(c) $\int \frac{dx}{x^2 - 3x + 2}$

(d) $\int \frac{x dx}{(x^2 - b^2)^2}$

(e) $\int \frac{dx}{(x^2 - 3)(x + 4)}$

(f) $\int \frac{x^2 dx}{(1-x)^{100}}$

(g) $\int \frac{dx}{\sqrt{x+1} + \sqrt{x-1}}$

(h) $\int \cos mx \cos nx dx$

(i) $\int \frac{\cos^4 x}{\sin^3 x} dx$

(j) $\int \frac{dx}{\sin^3 x \cos^5 x}$

(k) $\int \frac{dx}{\sin x \cos^4 x}$

(l) $\int \sin 5x \cos x dx$

(m) $\int (18x - 4)\sqrt{ax + bx} dx$

(n) $\int (18x - 6)\sqrt{-9x^2 + 6x} dx$

(o) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx$

(p) $\int \sin x \sin \frac{x}{2} \sin \frac{x}{3} dx$

(q) $\int \frac{dx}{\sin(x+a) \sin(x+b)}$

(r) $\int \tan x \tan(x+a) dx$